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AUTHOR Ellett, Frederick S., Jr.; Ericson, David P.
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ABSTRACT

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Probabilistic Extensions of the Traditional Forms of Path Analysis and Causal Modeling

Frederick S. Ellett, Jr.
David P. Ericson

Graduate School of Education
University of California, Los Angeles
405 Hilgard Avenue
Los Angeles, CA 90024

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Abstract

Several steps are taken to develop methods for analyzing systems that involve probabilistic causation. The basic ideas and distinctions are illustrated for systems with dichotomous variables. It is shown that these basic ideas have analogous counterparts in causal systems with continuous variables. By using a generalized conditional probability density function, it is shown that the causal models developed by proponents of path analysis and structural equation methods are special cases of a general probabilistic causal model. It is also argued that the general probabilistic causal model may be appropriate in situations where the traditional path analysis model does not fit. Implications for hypothesis testing are drawn.

Probabilistic Extensions of the Traditional Forms of
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I. Introduction

Causal modeling is quickly growing in popularity among researchers in the social and behavioral sciences. Already a well-accepted heuristic and analytic instrument in economics, sociology, and biology, causal modeling is now being strongly promoted in psychology and education as well. For example, see Bentler (1980) for an overview of past and recent developments. Causal modeling techniques travel under a variety of names--linear structural equation analysis, simultaneous equation methodology, path analysis, dependence analysis, covariance structural analysis, and simply, structural analysis. For many the intent is to model the causal relationships between variables so as to obtain the best "fit" with the data. Included in this are the tasks of identifying relevant variables and determining the direction of causality between those variables. Furthermore, especially in the case of path analysis and structural equation methods, the attempt is made to estimate the numerical value of the coefficients once a model has been specified.

We believe that the basic insights of this set of approaches to causal analysis, however, can be extended to cover systems which involve probabilistic causation. Several noted researchers, for example Cook and Campbell (1979), Cronbach (1980), and Suppes (1970), have argued that psychological systems should be analyzed in terms of probabilistic causation. Nonetheless, little has been done to develop the appropriate methodologies. We take steps toward developing these methods by considering systems with dichotomous variables and systems with continuous

variables. In this paper we will draw the distinction between deterministic and probabilistic causal systems. Then we will show how the basic methods of path analysis and structural equation methods may be extended to analyze such causal systems. One of the significant results of this research is that for every deterministic system which adequately fits the data there corresponds a probabilistic system which equally adequately fits the data. We shall also argue that the general probabilistic causal model may be appropriate in situations where the traditional path analysis model does not fit. In doing so, we shall draw implications for hypothesis testing.

II. Probabilistic Causation for a Causal System with Dichotomous Variables

In order to prepare the conceptual foundations for the analysis of causal systems with continuous variables, it will be helpful to consider how probabilistic causation appears in causal systems with dichotomous variables. The logically central idea here is that an idealized coin toss is the model of a probabilistic cause. Thus, as we often teach in our courses on probability, the tossing of a coin (T) has the probability p of producing a head and the probability (1 - p) of producing a tail. This is an idealized coin toss because it is assumed no other factors are present which affect the probability distributions of the outcome. Modern quantum physics, of course, can be interpreted as saying that the ultimate material of the world behaves in a purely probabilistic or stochastic manner. On the other hand, to say that C is a sufficient cause for an event E is to say that C is a deterministic cause of E. If C is sufficient for E, then the probability of E given C

equals 1. We'll return to the probabilistic/deterministic distinction later on.

It is well known that such writers as Simon, Blalock, Asher, and Duncan do apply their analytical procedures to causal systems with continuous or quantitative variables. Several of these writers, for example Asher (1976), Blalock (1964), and Davis (1975), have held that the basic results also hold for causal systems with dichotomous variables. Here we will briefly outline the general framework and present the equations which we have derived for these systems.

First, let us introduce the notation used in dichotomous situations. We'll assume, (1) that A is probabilistic cause of B (with probability (b + d)), (2) that B is a probabilistic cause of C (with probability f), and (?) that A is probabilistic cause of C (with probability (a + b)). We also assume that there are other, outside causes, S of B, T of C, and where A equals R, which have as their probabilities, respectively, P_2, P_3, and P_1. Using conditional probabilities, this causal system can be given the following, rather precise specification. First, $P(B.C/A.\bar{S}.\bar{T}) = b+df$, $P(B.\bar{C}/A.\bar{S}.\bar{T}) = d(1-f)$, $P(\bar{B}.C/A.\bar{S}.\bar{T}) = a$, and $P(\bar{B}.\bar{C}/A.\bar{S}.\bar{T}) = c$. The nonnegative numbers a, b, c, and d whose sum equals one give a complete specification of the common cause A's direct effects on B and C. When f equals zero, the previous four conditional probabilities reveal A's direct effects on B and C. Also, $P(C/B.\bar{A}.\bar{T}) = f$, $P(B/A.\bar{S}) = b + d$, and $P(C/A.\bar{S}.\bar{T}) = a + b+df$. Finally, $P(B/\bar{A}.S) = 1$ and $P(C/\bar{A}.\bar{B}.T) = 1$. Second, let us suppose, as Blalock and Asher customarily do, that R, S, and T are causally unrelated and pairwise uncorrelated. Given that the pairwise correlations of the "outside causes" are zero, we can show the following:

(1.0) $\emptyset_{AB} \cdot \sigma_B/\sigma_A = P(B/A) - P(B/\bar{A}) = (b+d)(1-P_2)$

(2.0) $\emptyset_{AC} \cdot \sigma_C/\sigma_A = (1-P_3)\,[a + b + df - (a+b+d)fP_2]$

(3.0) $\emptyset_{BC} \cdot \sigma_C/\sigma_B = [(1-a+b+d)P_1)(P_2f - (a+b+d)P_1P_2f+fP_1d+aP_1P_2+bP_1)$
$- aP_1(1-f)(P_2+dP_1 - (a+b+d)P_1P_2)] \cdot [(1-P_2)(1-P_3)]$
divided by $[P_2+(b+d)P_1(1-P_2)]\,[1-P_2-(b+d)P_1(1-P_2)]$.

There are several important results which follow from these equations. First, when we have only a simple causal chain from A to B to C, that is when a + b = 0, it follows that $\emptyset_{AC} = \emptyset_{AB}\emptyset_{BC}$. Second, when we have only a common cause from A to B and to C, that is when f = 0, it can be shown that $\emptyset_{BC} \gtreqless \emptyset_{AB}\emptyset_{AC}$ if and only if $b - (a+b)(b+d) \gtreqless 0$. Notice that if A were to cause B and to cause C, but never to cause both B and C at the same time, then b = 0 and will not equal (a+b)(b+d). Hence, $\emptyset_{BC} \neq \emptyset_{BA}\emptyset_{AC}$ even though A is the only common causal factor around.

Also notice that $\emptyset_{AB} \cdot (\sigma_B/\sigma_A) = (b+d)(1-P_2)$. Although this coefficient will be <u>non-negative</u> whenever $(b+d) > 0$, the coefficient need not equal $P(B/A.\bar{S})$, which equals b + d. In this causal system, it is the conditional probability $P(B/A.\bar{S})$ which adequately characterizes the degree of A's causal influence on B. Given these equations, it can also be shown that the path analysis and structural equation coefficient $(\sigma_C/\sigma_B) \cdot (\emptyset_{BC} - \emptyset_{AB} \cdot \emptyset_{AC})/(1-\emptyset_{AB}^2)$, which is used to give the degree of B's (direct) causal influence on C for systems with continuous variables, will not give the proper results for systems with dichotomous variables.

It is useful to draw the distinction between a pure-and-system and a pure-or-system. This distinction is related to the way the common cause A directly affects B and C. In a pure-and-system, the cause A acts as if it were a double-coin toss in which the first coin has a

probability of (b + d) of coming up B and the second coin has a probability of (a + b) of coming up C. In such a situation, it turns out that b = (a+b)(b+d). On the other hand, in a pure-or-system, the cause A never leads to both B and C on any given occasion. Cartwright (1979) and Salmon (1980) have recently considered situations in the physical sciences where the common cause A of B and C has such features. Salmon, for example, considers the situation where an (excited) electron is in the 4th energy level of an atom, but where the electron can go either to the 3rd energy level with a certain probability or to the 2nd energy level with a certain probability, yet it can not go to both the 2nd and 3rd energy levels. For social scientists, the following kind of situation might be more interesting. Suppose that A is a probabilistic cause of B and B is a probabilistic cause of C. Suppose also that A is a probabilistic cause of D and D is a probabilistic cause of C. In this kind of situation, there are two causal paths from A to C. Now suppose that A is a pure-or common cause of B and D. In particular, let A be the randomized assignment of individuals either to B (treatment B) or to C (treatment C). Then the causal system being analyzed is a randomized experiment with two treatment groups. (As we noted above, for this kind of causal system the term $\emptyset_{AB}\ (\sigma_B/\sigma_A)$ in equation 1.0 will be nonnegative if $b+d > 0$, and so it can be used to test whether A has any causal influence on B. Difficulties do arise, however, when one tries to check the value of $\emptyset_{BC}\ (\sigma_C/\sigma_B)$, for its value depends on the unknown values of P_2 and P_3. Although we can't go into the argument here, we believe such difficulties can only be resolved if one develops a way of testing the conditional probabilities which give the precise specification to the model.)

Thus, not only can causal systems be analyzed where probabilistic causation is involved, but a wide variety of common causes A can be studied by changing the parameters a, b, c, and d. One is not limited to systems in which A is a pure-and common cause of B and C; one can also investigate systems in which A is a pure-or common cause of B and C. The pure-and causal system and the pure-or causal system are but special cases of the general causal system where the common cause A is characterized by a, b, d, and d.

III. Probabilistic Causation for a Causal System with Continuous Variables

Given this basic understanding of probabilistic causation in a system with dichotomous variables, we can now consider causal systems with continuous variables. Let us assume that R, S, and T are continuous variables which are causally unrelated and which have a well-defined joint probability density function $p(r,s,t)$ with finite means (ER, ES, and ET) and finite variances $\sigma(R,R)$, $\sigma(S,S)$, and $\sigma(T,T)$. Let us also assume that the three covariances $\sigma(R,S)$, $\sigma(R,T)$, and $\sigma(S,T)$ are zero.

Let X, Y, and Z be the factors constituting the system under investigation. Assume that these factors are continuous variables such that the variables R, S, T, X, Y, and Z have a joint probability density function of the following form:

$$w(x,y,z,r,s,t) = w(x,y,z|r,s,t)p(r,s,t),$$

where $w(x,y,z\ r,s,t)$ is defined as follows:

(4.1) $\quad w(x = r,\ y = s,\ z = t|r,s,t) = c\cdot(1-f)$

(4.2) $\quad w(x = r,\ y = s,\ z = Bs + t|r,s,t) = c\cdot f$

(4.3) $w(x = r,\ y = s,\ z = Cr + t|r,s,t) = a(1-f)$

(4.4) $w(x = r,\ y = s,\ z = Cr + Bs + t|r,s,t) = af$

(4.5) $w(x = r,\ y = Ar + s,\ z = Cr + t|r,s,t) = b(1-f)$

(4.6) $w(x = r,\ y = Ar + s,\ z = Cr + ABr + Bs + t|r,s,t) = bf$

(4.7) $w(x = r,\ y = Ar + s,\ z = t|r,s,t) = e(1-f)$

(4.8) and $w(x = r,\ y = Ar + s,\ z = ABr + Bs + t|r,s,t) = ef.$

If $p(r,s,t)$ is zero, then $w(x,y,z|r,s,t)$ is also defined to be zero; the probability parameters a, b, c, e, and f are nonnegative numbers such that $a + b + c + e = 1$ and $0 \leq f \leq 1$. There are no restrictions on the "multipliers," A, B, or C.

The conditional probability density $w(x,y,z\ r,s,t)$ can, of course, be specified by means of the joint generalized probability density function. (Doob, 1953; Feller, 1957; Loeve, 1960, Parzen, 1960). Here the generalized probability density function has the following definition:

(5) $w(x,y,z,r,s,t) =$

$$
\begin{aligned}
&[c(1-f)\cdot\delta(x = r,\ y = s, z = t)\\
&\quad + cf\cdot\delta(x = r, y = s, z = Bs + t)\\
&\quad + a(1-f)\cdot\delta(x = r, y = s, z = Cr + t)\\
&+\ af\cdot\delta(x = r, y = s, z = Cr + Bs + t)\\
&+\ [b(1-f)\cdot\delta(x = r, y = Ar + s, z = Cr + t)\\
&\quad + bf\cdot\delta(x = r, y = Ar + s, z = Cr + ABr + Bs + t)\\
&\quad + e(1-f)\cdot\delta(x = r, y = Ar + s, z = t)\\
&+\ ef\cdot\delta(x = r, y = Ar + s, z = ABr + Bs + t)]\cdot p(r,s,t).
\end{aligned}
$$

Kronecker's delta function, $\delta(x = r, y = s, z = t)$, takes on the value 1 if $x = r, y = s$, and $z = t$ and which is zero otherwise. In this causal system, X is a common cause of Y and Z and Y is a cause of Z.

We now present the derivation of some mathematical relations for the general class of causal models described by equation (5). The arguments rely on the fact that for random variables with finite means and second moments:

(i) $E_V(V) = E_W E_V(V|W)$,

(ii) $\text{var}_V(V) = E_W E_V(V|W) - (E_V(V))^2$ and

(iii) $\text{cov}_{U,V}(U,V) = E_W[\text{cov}_{U,V}(U,V|W)] - E_U(U)E_V(V)$.

Claim 1. $EX = ER$ and $\sigma(X,X) = \sigma(R,R)$.

Proof: These results follow directly from the fact that $X = R$ for all values of R.

Claim 2. $EY = (b + e)A \cdot ER + ES$.

Proof: $E(Y) = E_{R,S,T}E_Y(Y|R,S,T) = E(AR+S)Pr(X=AR+S) + E(S)Pr(X=S)$

$= E(AR)Pr(X=AR+S) + E(S) = (b+e)A \cdot E(R) + E(S)$.

Claim 3. $EZ = (a+b)(C) \cdot ER + (b+c)fA \cdot B \cdot ER + fB \cdot ES + ET$.

Proof: $E(Z) = E_{R,S,T}E_Z(Z|R,S,T) = E_{R,T}(CR+T)[b(1-f) + a(1-f)]$

$+ E_{R,S,T}(CR+BS+T)af + E_{R,S,T}[(C+AB)R + BS+T]bf$

$+ E_{S,T}(BS+T)cf + E_T(T)[c(1-f) + e(1-f)]$

$= (a+b)C \cdot E(R) + (b+e)fA \cdot B \cdot E(R) + fB \cdot E(S) + E(T)$.

Claim 4. $\sigma(Y,Y) = \sigma(S,S) + (b+e)A^2 \cdot ER^2 - (b+e)^2A^2 \cdot (ER)^2$.

Proof: $\sigma(Y,Y) = E_{R,S,T}E_Y(Y^2|R,S,T) - (E_Y(Y))^2$

$= E_{R,S}(AR+S)^2(b+e) + E_S(S)(a+c)$

$- [(b+e)A \cdot E(R) + E(S)]^2$

$= (b+e)A^2E(R^2) + 2A(b+e)E(RS) + E(S^2)$

$- (b+e)^2A^2(E(R))^2 - 2A(b+e)E(R)E(S) - (E(S))^2$

$$= \sigma(S,S) + (b+e)A^2E(R^2) - (b+e)^2A^2(ER)^2$$
$$- 2A(b+e)\mathrm{Cov}(R,S).$$

Since $\mathrm{Cov}(R,S) = 0$ by assumption, the claim is proved.

Claim 5. $\sigma(X,Y) = (b+e)A\cdot\sigma(X,X)$.

Proof: $\sigma(X,Y) = E_{R,S,T}E(XY|R,S,T) - E_X(X)E_Y(Y)$

$$= E_{R,S}(R\cdot(AR+S))(b+e) + E_{R,S}(RS)(a+c)$$
$$- (ER)((b+e)A\cdot ER+ES)$$
$$= (b+e)A\cdot E_R(R^2) + E(RS) - (b+e)A\cdot(ER)^2 - (ER)(ES)$$
$$= (b+e)A\cdot\sigma(R,R) + \mathrm{Cov}(R,S).$$

Since $\sigma(R,R)$ equals $\sigma(X,X)$ by claim 1 and since $\mathrm{Cov}(R,S,) = 0$ by assumption, the claim is proved.

Claim 6. $\sigma(X,Z) = [(b+e)fAB + (a+b)C]\cdot\sigma(X,X)$.

Proof: $\sigma(X,Z) = E_{R,S,T}E(XZ|R,S,T) - E_X(X)E_Z(Z)$

$$= E_R(CR^2)(a+b) + E_{R,S,T}(R(BS+T))f + E_R(ABR^2)(b+e)f$$
$$+ E_{R,T}(RT)(1-f) - (ER)\cdot[(a+b)C\cdot E(R)$$
$$+ (b+e)fAB\cdot E(R) + fB\cdot E(S) + E(T)]$$
$$= (a+b)C[E(R^2) - (ER)^2] + fB[E(RS) - (ER)(ES)]$$
$$+ E(RT) - E(R)E(T) + (b+e)fAB[E(R^2) - (ER)^2]$$
$$= [(b+e)fAB + (a+b)C]\cdot\sigma(R,R) + fB\mathrm{Cov}(R,S) + \mathrm{Cov}(R,T).$$

Since $\sigma(R,R)$ equals $\sigma(X,X)$ and since $\mathrm{Cov}(R,S) = 0$ and $\mathrm{Cov}(R,T) = 0$ by assumption, the claim is proved.

Claim 7. $\sigma(Y,Z) = fB\cdot\sigma(Y,Y) + AC\cdot[bE(X^2) - (a+b)(b+e)(EX)^2]$.

Proof: $\sigma(Y,Z) = E_{R,S,T}E(YZ|R,S,T) - E_Y(Y)E_Z(Z)$

$$= E_{R,S,T}((AR+S)(CR+T))b + E_{R,S}((AR+S)(ABR+BS))(b+e)f$$
$$+ E_{R,S,T}((AR+S)T)e + E_{R,S,T}(S(CR+T))a$$
$$+ (a+c)fBE_S(S^2) + E_{S,T}(ST)c$$

$$- [(b+e)AE(R) + E(S)][(a+b)CE(R) + (b+e)fABE(R)$$
$$+ fBE(S) + E(T)]$$
$$= fB[E(S^2) - (E(S))^2 + (b+e)A^2E(R^2) - (b+e)^2A^2(ER)^2]$$
$$+ bAC \cdot E(R)^2 - (a+b)(b+e)AC(ER)^2$$
$$+ [2(b+e)fAB + (a+b)C][E(RS) - (ER)(ES)]$$
$$+ E(ST) - E(S)E(T) + (b+e)A[E(RT) - E(S)E(T)]$$
$$= fB[E(S^2) - (E(S))^2 + (b+e)A^2E(R^2) - (b+e)^2A^2(ER)^2]$$
$$+ bAC \cdot E(R)^2 - (a+b)(b+e)AC(ER)^2$$
$$+ [2(b+e)fAB + (a+b)C]Cov(R,S)$$
$$+ Cov(S,T) + (b+e)A\ Cov(R,T).$$

Since the term inside the brackets equals $\sigma(Y,Y)$ by claim 4 and since $Cov(R,S) = Cov(S,T) = Cov(R,T) = 0$ by assumption, the claim is proved.

IV. Interpretations of the General Model

We are now prepared to give interpretations to the general class of causal models which is represented in figure 1. Suppose that R,S, and T are the outside causal factors (or error terms) which are causally unrelated and pairwise uncorrelated and that the system consisting of X,Y, and Z is causally deterministic. The system is causally deterministic when $b = 1$ and $f = 1$. Thus, in a causally deterministic system, the joint conditional probability density has the following form:

$$w(x = r,\ y = \bar{A}r + s, z = (\bar{A} \cdot \bar{B} + \bar{C})r + \bar{B}s + t | r,s,t) = 1.$$

(The parameters $\bar{A}$, $\bar{B}$, and $\bar{C}$ are used to characterize a causally deterministic system.) Alternatively, the joint generalized probability density function becomes

$w(x,y,z,r,s,t) = \delta(x = r, y = \bar{A}r + s, z = (\bar{A} \cdot \bar{B} + \bar{C})r + Bs + t) \cdot p(r,s,t)$.

In such a system, the limiting case where both b and f go to 1, we get corollary 1:

(6.1) $\sigma(X,Y) = \bar{A} \cdot \sigma(X,X)$,

(6.2) $\sigma(X,Z) = (\bar{A} \cdot \bar{B} + \bar{C}) \cdot \sigma(X,X)$,

(6.3) and $\sigma(Y,Z) = \bar{B} \cdot \sigma(Y,Y) + \bar{A} \cdot \bar{C} \cdot \sigma(X,X)$.

(See Figure 2 for the standard representation of a causally deterministic interpretation.) Notice that these equations are identical to the basic equations of the path analytic and the structural equation approaches for a causally deterministic system where X directly causes Y and Y directly causes Z, and so X is an indirect cause of Z, and also where X is also a direct cause of Z. (Asher, 1976; Blalock, 1971; Duncan, 1971, 1975; Kenny, 1979; Wright, 1971.)

Suppose now we consider other interpretations than the "limiting" case of causal determinism. Heuristically, instead of a causal factor being sufficient for its (part of the) effect, let us consider cases where it confers a probability on the effect. For example, in a causally deterministic system, for any individual it is true that $Y_i = AX_i + S_i$; in a probabilistic system, it follows that $P(Y_i = AX_i + S_i | X_i \text{ and } S_i) = b + e$, which need not equal 1. Suppose, then, that the parameters a,b,c,e, and f are interpreted as parameters of probabilistic causation. (See Good, 1961, 1962; Salmon, 1980; Suppes, 1970.) Again, there should be no difficulty about whether such a conception is intelligible. In quantum physics, there is already a widespread use of probabilistic or (stochastic) models to analyze apparently undeterministic phenomena such as small particle physical phenomena. Given the nature of psychological phenomena, we can expect a growing realization of the need for models,

such as those introduced here, capable of handling probabilistic causation. Several noted writers have, of course, recognized such a need. The earlier work of Suppes (1970) and the recent works of Cook and Campbell (1979; Ch.1) and Cronbach (1980, pp. 20-21) illustrate this growing recognition.

In section II we illustrated how probabilistic causal systems with dichotomous variables could be analyzed. In our discussion of such causal systems, we drew the distinction between a pure-and-system and a pure-or-system. In an analogous manner, it can be seen that the common cause X of Y and Z can be characterized by the parameters a, b, c, e such that the causal system with continuous variables can be a pure-and-system when $b = (a+b)(b+e)$ or it can be a pure-or-system when $b = 0$. In a pure-and-system, we get corollary 2:

(7.1) $\sigma(X,Y) = (b + e)A \cdot \sigma(X,X)$,

(7.2) $\sigma(X,Z) = [(b + e)AfB + (a + b)C] \cdot \sigma(X,X)$,

(7.3) and $\sigma(Y,Z) = fB \cdot \sigma(Y,Y) + (a + b)(b + e)AC \cdot \sigma(X,X)$.

In a pure-or-system, we get corollary 3:

(8.1) $\sigma(X,Y) = (1-a)A \cdot \sigma(X,X)$,

(8.2) $\sigma(X,Z) = [(1-a)AfB + aC] \cdot \sigma(X,X)$,

(8.3) and $\sigma(Y,Z) = fB \cdot \sigma(Y,Y) - a(1-a)AC \cdot (EX)^2$.

Notice that even if the multipliers A,B, and C are positive, it is possible for $\sigma(Y,Z)$ to be negative in a pure-or-system. And just as in the case of causal systems with dichotomous variables, the parameters a, b, c, and e enable one to consider a wide range of ways in which the common cause X directly influences Y and Z.

Undecidability Claim: Given the population parameters $\sigma(X,X)$, $\sigma(X,Y)$, $\sigma(X,Z)$, $\sigma(Y,Y)$, and $\sigma(Y,Z)$, it is empirically impossible to decide between a causally deterministic model and a probabilistic pure-and-system, where both fit the population parameters.

Proof: According to corollary 1, if a causally deterministic system fits the population parameters then there exists $\bar{A}$, $\bar{B}$, and $\bar{C}$ such that

(9.1) $\sigma(X,Y) = \bar{A} \cdot \sigma(X,X)$,

(9.2) $\sigma(X,Z) = [\bar{A}\bar{B} + \bar{C}] \cdot \sigma(X,X)$,

(9.3) $\sigma(Y,Z) = \bar{B} \cdot \sigma(Y,Y) + \bar{A}\bar{C} \cdot \sigma(X,X)$.

But then a probabilistic pure-and system can be postulated which will also fit the population parameters. According to corollaries 1 and 2, merely find a,b,c,e,f, and A,B, and C for a probabilistic system such that

(9.4) $(b + e)A = \bar{A}$,

(9.5) $fB = \bar{B}$,

(9.6) and $(a + b)C = \bar{C}$.

Conversely, suppose that there exists a probabilistic pure-and-system which fits the populaton parameters. Thus, according to corollary 2,

(9.7) $\sigma(X,Y) = (b + e)A \cdot \sigma(X,X)$,

(9.8) $\sigma(X,Z) = [(b + e)AfB + (a + b)C] \cdot \sigma(X,X)$,

(9.9) and $\sigma(Y,Z) = fB \cdot \sigma(Y,Y) + (a + b)C(b + e)A \cdot \sigma(X,X)$.

But then there exists a causally deterministic system which also fits the population parameters. Using corollaries 1 and 2, merely let the causal systems indices $\bar{A}$, $\bar{B}$, and $\bar{C}$ be such that

(9.10) $\bar{A} = (b + e)A$,

(9.11) $\bar{B} = fB$,

(9.12) and $\bar{C} = (a + b)C$.

The importance of the undeciability claim lies in the fact that it shows that additional kinds of data are required before one can make a causal interpretation of one of the causal models. Cook and Campbell (1979, p. 22) have noted that,

> [A]s practicing scientists, we in fact pay little or no attention to the merely logical possibility of alternative theories [which also fit the data] . . . Toulmin (1961, pp. 113-15) has stated the point well:
>
> > Again, philosophers sometimes assert that a finite set of empirical observations can always be explained in terms of an infinite number of hypotheses. The basis for this remark is the simple observation that through any finite set of points an infinite number of mathematical curves can be constructed. If there were no more to "explanation" than curve fitting, this doctrine would have some bearing on scientific practice. In fact, the scientist's problem is very different: In an intellectual situation which presents a variety of demands, his or her task is--typically--to accommodate some new discovery to the scientist's own inherited ideas, without needlessly jeopardizing predecessors' intellectual gains. This kind of problem has an order of complexity quite different from that of simple curve fitting: Far from a scientist having an infinite number of possibilities to choose between, it may be a stroke of genius to imagine even a single one.
>
> It is only when there exist actually developed alternative explanations . . . that validity questions arise for theories and other causal hypotheses whose predictions have been confirmed.

The probabilistic causal system provides a real alternative, not just a logically possible one, to the deterministic model. And since a

causally deterministic and a probabilistic causal system equally well fit the covariance and variance data, the inferences to the underlying form of causal processes and the degrees of causal influence are blocked. Additional kinds of evidence and data must be gathered in order to decide between the two causal models. This, of course, points to a new kind of interpretative issue for causal modeling. For if the covariance and variance data themselves cannot settle our choice of models, we shall need to develop a new theory of model determination. Though this task is beyond the scope of this paper, attention to the following may prove helpful in seeking such resolve. Focus upon the distributions of variables within the model may help determine matters; further, greater specification and elaboration of the error terms and their relations with the other variables is likely to be of great importance (and in the case of categorical variables perhaps all that is needed). Alternatively, our theory of the causal system under examination will doubtless play an important role. In this regard, Bayesian approaches to the problem of inference may have much to offer. Given this mixture of empirical and theoretical considerations, the problem of deciding between deterministic and probabilistic models should not prove obstinate.

On the other hand, there are additional advantages of the general probabilistic causal model. For in those situations where a deterministic causal model (and hence a probabilistic pure-and-system) does not adequately fit the data, it could turn out that a probabilistic system still could be found which adequately fits the data by varying the parameters a, b, c, e of the common cause X. Hence, it is possible, indeed likely, that the general probabilistic model will be appropriate in more situations than the deterministic model.

IV. Other Causal Systems the Model Fits

We would like to point out in closing that other important causally deterministic models can also be "Probabilified." In figure 4 we present the general path analysis or structural equation model of the relations between attitudes and behavior presented in a recent review article by Peter Bentler (1980), where $\sigma(X_1,V_1) = \sigma(X_2,V_1) = \sigma(X_3,V_1) = \sigma(X_1,V_2) = \sigma(X_2,V_2) = \sigma(X_3,V_2) = \sigma(V_1,V_2) = 0$. For such a system one can derive the following path/structural equations.

(10.1) $\sigma(X_1,Y_1) = \bar{A}\cdot\sigma(X_1,X_1) + \bar{B}\cdot\sigma(X_1,X_2) + \bar{R}\cdot\sigma(X_1,X_3)$,

(10.2) $\sigma(X_2,Y_1) = \bar{A}\cdot\sigma(X_1,X_2) + \bar{B}\cdot\sigma(X_2,X_2) + \bar{R}\cdot\sigma(X_2,X_3)$,

(10.3) $\sigma(X_3,Y_1) = \bar{A}\cdot\sigma(X_1,X_3) + \bar{B}\cdot\sigma(X_3,X_2) + \bar{R}\cdot\sigma(X_3,X_3)$,

(10.4) $\sigma(X_1,Y_2) = (\bar{D} + \bar{C}\bar{A})\cdot\sigma(X_1,X_1) + \bar{C}\bar{B}\cdot\sigma(X_1,X_2) + (\bar{S} + \bar{C}\bar{R})\cdot\sigma(X_1,X_3)$,

(10.5) $\sigma(X_2,Y_2) = (\bar{D} + \bar{C}\bar{A})\cdot\sigma(X_1,X_2) + \bar{C}\bar{B}\cdot\sigma(X_2,X_2) + (\bar{S} + \bar{C}\bar{R})\cdot\sigma(X_2,X_3)$,

(10.6) $\sigma(X_3,Y_2) = (\bar{D} + \bar{C}\bar{A})\cdot\sigma(X_1,X_3) + \bar{C}\bar{B}\cdot\sigma(X_3,X_2) + (\bar{S} + \bar{C}\bar{R})\cdot\sigma(X_3,X_3)$,

(10.7) and $\sigma(Y_1,Y_2) = (\bar{D}\bar{A})\cdot\sigma(X_1,X_1) + \bar{D}\bar{B}\cdot\sigma(X_1,X_2) + \bar{C}\cdot\sigma(Y_1,Y_1)$
$+ (\bar{S}\bar{A} + \bar{D}\bar{R})\cdot\sigma(X_3,X_1) + \bar{S}\bar{B}\cdot\sigma(X_2,X_3) + \bar{S}\bar{R}\cdot\sigma(X_3,X_3)$

Given an analogous specification of the probability parameters as in figures 1, 2 and 3, one can derive the following equations for a probabilistic pure-and-system (see figure 5):

(11.1) $\sigma(X_1,Y_1) = (b + d)A\cdot\sigma(X_1,X_1) + fB\cdot\sigma(X_1,X_2), + (q + s)R\cdot\sigma(X_1,X_3)$,

(11.2) $\sigma(X_2,Y_1) + (b + d)A\cdot\sigma(X_1,X_2) + fB\cdot\sigma(X_2,X_2), + (q + s)R\cdot\sigma(X_2,X_3)$

(1.13) $\sigma(X_3,Y_1) = (b + d)A\cdot\sigma(X_1,X_3) + fB\cdot\sigma(X_3,X_2) + (q + s)R\cdot\sigma(X_3,X_3)$,

(11.4) $\sigma(X_1,Y_2) = [(a + b)D + (b + d)gAC]\cdot\sigma(X_1,X_1) + fgBC\cdot\sigma(X_1,X_2)$
$+ [(q + s)gCR + (p + q)S]\cdot\sigma(X_1,X_3)$,

(11.5) $\sigma(X_2,Y_2) = [(a + b)D + (b + d)gAC]\cdot\sigma(X_1,X_2) + fgBC\cdot\sigma(X_2,X_2)$
$+ [(q + s)gCR + (p + q)S]\cdot\sigma(X_2,X_3)$,

$$(11.6)\quad \sigma(X_3,Y_2) = (a + b)D + (b + d)gAC]\sigma(X_1,X_3) + fgBC\cdot\sigma(X_2,X_3) + [(q + s)gCR + (p + q)S]\cdot\sigma(X_3,X_3),$$

$$(11.7)\quad \sigma(Y_1,Y_2) = (a + b)(b + d)AD\cdot\sigma(X_1,X_1) + (a + b)fDB\cdot\sigma(X_1,X_2) + gC\cdot\sigma(Y_1,Y_1) + [(a + b)(q + s)DR + AS(b + d)(p + q)]\cdot\sigma(X_1,X_3) + f(q + p)BS\cdot\sigma(X_2,X_3) + (p + q)(q + s)RS\cdot\sigma(X_3,X_3).$$

Given only the values of the covariances, the causally deterministic system is empirically indistinguishable from the probabilistic pure-and-system when

$$(12.1)\quad \bar{A} = (b + d)A,$$

$$(12.2)\quad \bar{B} = fB,$$

$$(12.3)\quad \bar{C} = gC,$$

$$(12.4)\quad \bar{D} = (a + b)D,$$

$$(12.5)\quad \bar{R} = (q + s)R,$$

$$(12.6)\quad \text{and}\quad \bar{S} = (p + q)S.$$

It can be shown that there are other kinds of causal systems which can be "probabilified." In figure 6 and in table 1, we present the relevant probabilistic representation and mathematical equations for the causal systems which O. Duncan analyzed in his book Introduction to Structural Equation Models (1975). In figure 7 and in table 2, we present the relevant probabilistic representation and mathematical equations for the causal system which A. Jensen put forward in Bias in Mental Testing (1980) to account for adult IQ(S) in terms of such causal factors as father's education (X), father's occupation (Y), child's education (Z), and early childhood IQ (R) (p.337). By taking the appropriate limits in the equations, the probabilistic model represented in figure 6 and the model represented in figure 7 can handle several of the

causal systems discussed by such writers as Asher in Causal Modeling (1976). (For example, see the Miller and Stokes representation, figure 14, p. 43 and figure 12, p. 39). At any rate, for each of these deterministic causal models, there exists a probabilistic pure-and-causal model which will fit the data as well as the deterministic model. On the other hand, in those situations where a causally deterministic system does not fit the data very well, it could turn out that a probabilistic causal model in which the common causes in the system work in non-pure-and fashion would provide a more adequate fit.

V. Summary

It is well known that several leading writers and researchers, such as Cook and Campbell (1979), Cronbach (1981), and Suppes (1970) have argued that an appropriate methodology should be developed to deal with causal systems in which probabilistic causation is involved.

In section II we have presented the basic ideas that are involved in analyzing systems with dichotomous variables. We have explicated the central notion of probabilistic causation, and for common causes we have drawn the distinction between a pure-and-cause and a pure-or-cause. We have explained how the methods might handle such diverse causal systems.

In section III we have shown how causal systems with continuous variables could be understood as systems involving probabilistic causation. In sections III and IV we argued that for certain kinds of causal systems, whenever a causally deterministic model adequately fits the data, a probabilistic causal model will also do so. We drew out some of the implications for causal hypothesis testing and inference related to

this fact. Finally, we argued that the general probability model may provide good fits in situations where the traditional path analysis or structural equation methods do not work well.

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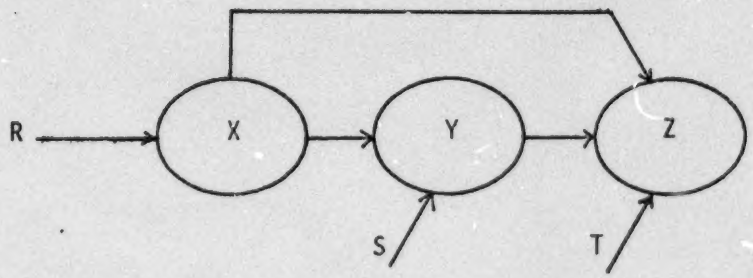
R
X
Y
Z
S
T

Figure 1: Representation of the General Causal Model in which X is a Common Cause of Y and Z and Y is a (Direct) Cause of Z.

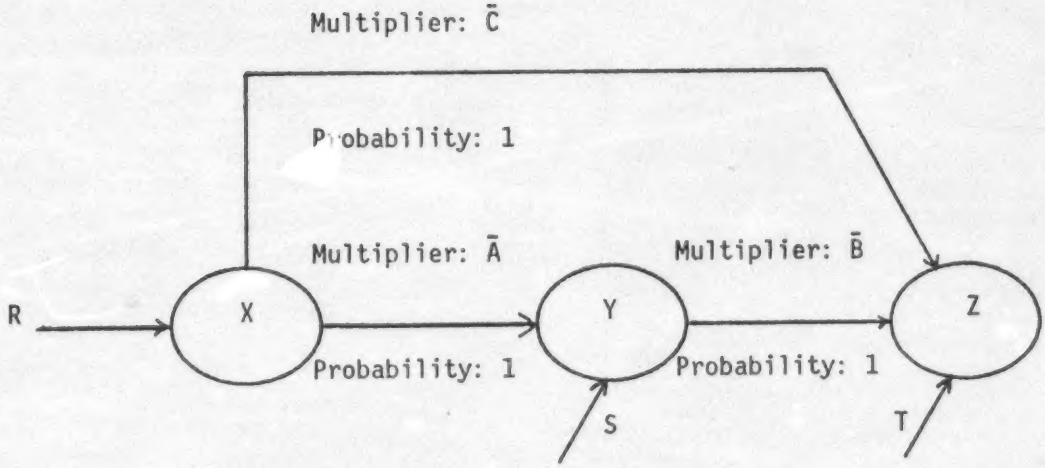
Multiplier: $\bar{C}$
Probability: 1
Multiplier: $\bar{A}$
Multiplier: $\bar{B}$
R
X
Y
Z
Probability: 1
Probability: 1
S
T

Figure 2: Representation of the Causally Deterministic Interpretation of the Model where X is a Common Cause of Y and Z and Y is a Direct Cause of Z.

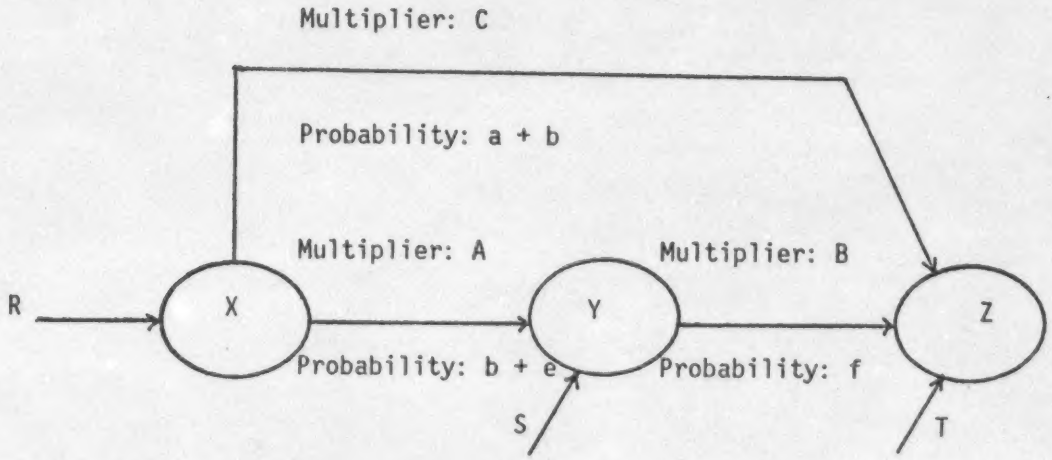
Multiplier: C
Probability: a + b
Multiplier: A
Multiplier: B
R
X
Y
Z
Probability: b + e
Probability: f
S
T

Figure 3: Representation of the Probabilistic Pure-and-System Interpretation of the Model in which X is a Common Probabilistic Cause of Y and Z and Y is a Probabilistic Cause of Z.

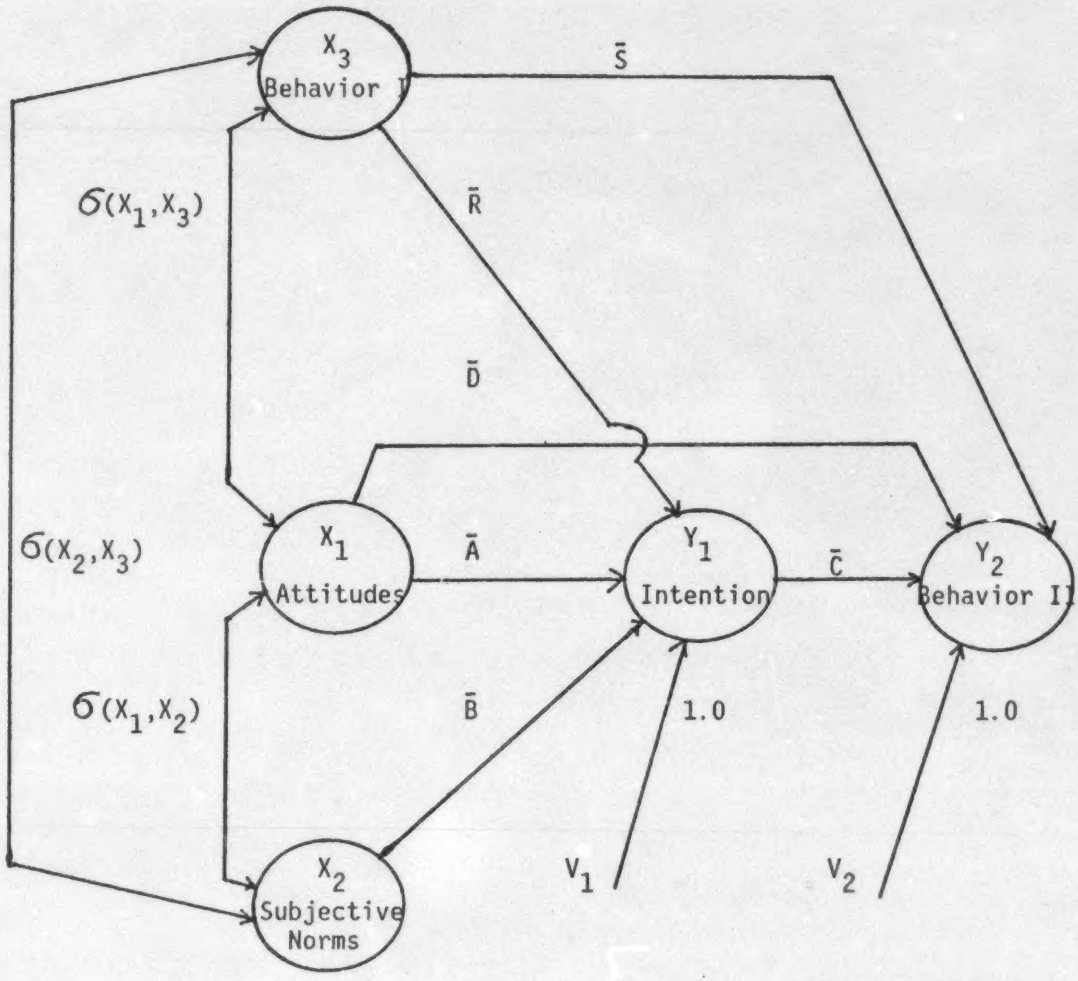
X_3
Behavior I
$\bar{S}$
$\sigma(X_1, X_3)$
$\bar{R}$
$\bar{D}$
$\sigma(X_2, X_3)$
X_1
Attitudes
$\bar{A}$
Y_1
Intention
$\bar{C}$
Y_2
Behavior II
$\sigma(X_1, X_2)$
$\bar{B}$
1.0
1.0
V_1
V_2
X_2
Subjective
Norms

Figure 4: Representation of the Causally Deterministic Interpretation of P. Bentler's Causal System (1980) for Motivation and Behavior.

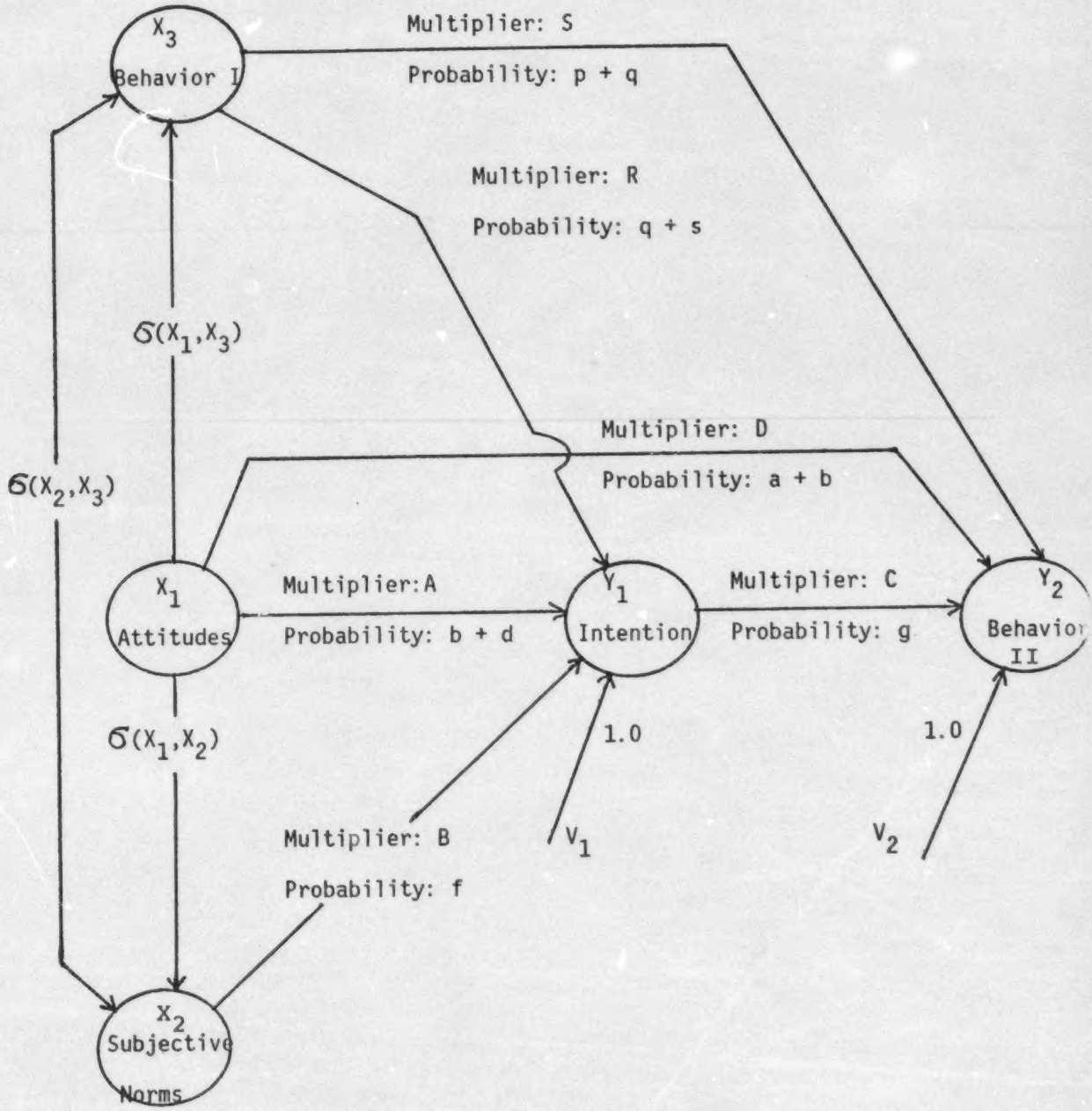
X_3
Behavior I
Multiplier: S
Probability: p + q
Multiplier: R
Probability: q + s
$\sigma(X_1, X_3)$
Multiplier: D
Probability: a + b
$\sigma(X_2, X_3)$
X_1
Attitudes
Multiplier:A
Probability: b + d
Y_1
Intention
Multiplier: C
Probability: g
Y_2
Behavior II
$\sigma(X_1, X_2)$
1.0
1.0
Multiplier: B
Probability: f
V_1
V_2
X_2
Subjective Norms

Figure 5: Representation of the Probabilistic Interpretation of P. Bentler's Causal System (1980) for Motivation and Behavior.

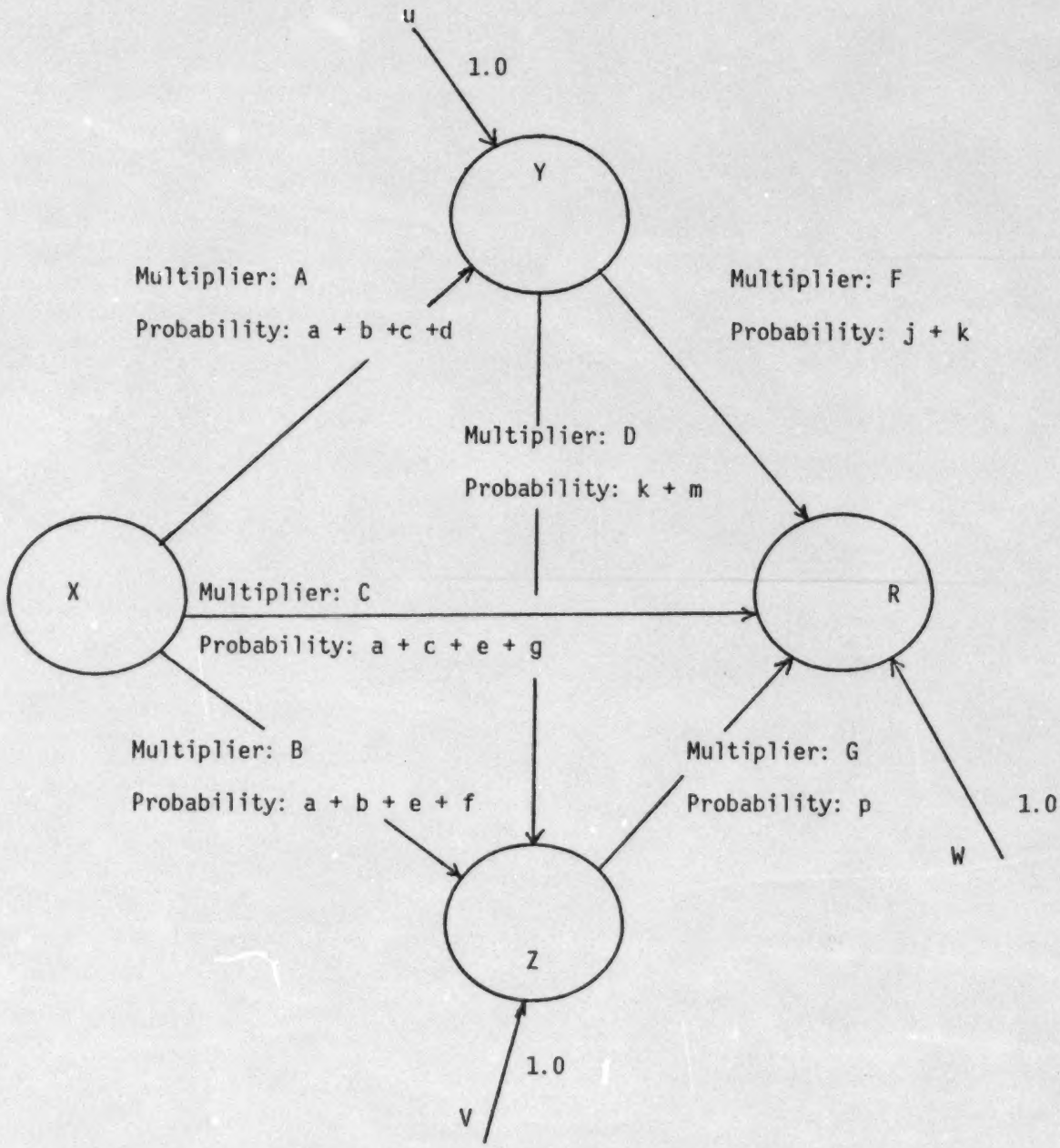
u
1.0
Y
Multiplier: A
Probability: a + b +c +d
Multiplier: F
Probability: j + k
Multiplier: D
Probability: k + m
X
Multiplier: C
Probability: a + c + e + g
R
Multiplier: B
Probability: a + b + e + f
Multiplier: G
Probability: p
1.0
W
Z
1.0
V

Figure 6: Presentation of the Probabilistic Interpretation of O. Duncan's Causal System (1975)

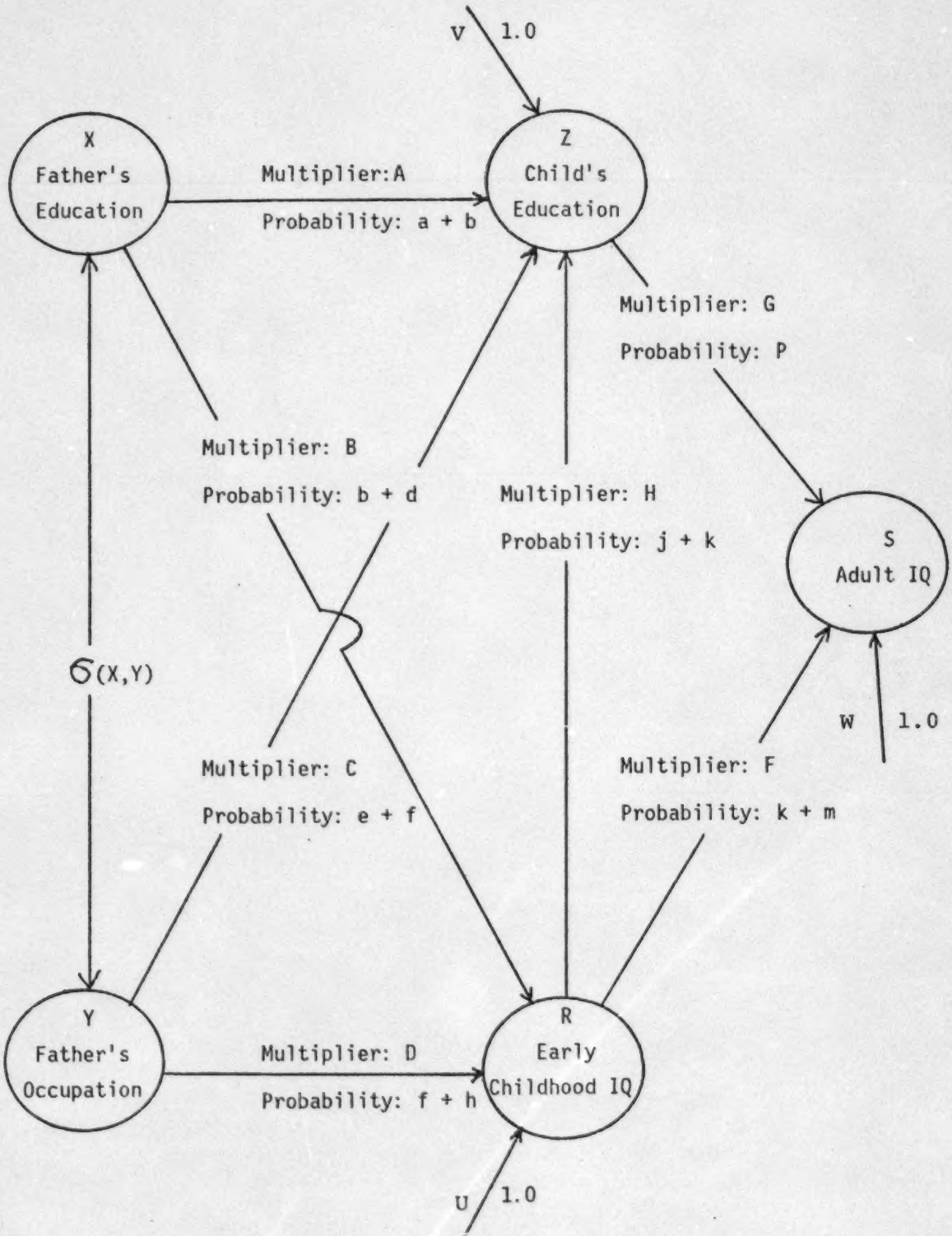
V
1.0
X
Father's
Education
Multiplier:A
Probability: a + b
Z
Child's
Education
Multiplier: G
Probability: P
Multiplier: B
Probability: b + d
Multiplier: H
Probability: j + k
S
Adult IQ
σ(X,Y)
W
1.0
Multiplier: C
Probability: e + f
Multiplier: F
Probability: k + m
Y
Father's
Occupation
Multiplier: D
Probability: f + h
R
Early
Childhood IQ
U
1.0

Figure 7: Representation of a Probabilistic Interpretation of A. Jensen's Causal System (1980, p. 337) for Adult IQ.

Table 1

The Mathematical Relations for the Probabilistic Interpretation of O. Duncan's Causal System (1975)

[Deterministic equations result when a = k = p = 1]

$$\sigma(X,Y) = (a + b + c + d)A \cdot \sigma(X,X)$$

$$\sigma(X,Z) = \left[(a + b + e + f)B + (a + b + c + d)(k + m)AD\right]\sigma(X,X)$$

$$\sigma(X,R) = [(a + c + e + g)C + (a + b + c + d)(j + k)AF$$
$$+ (a + b + e + f)pBG + (a + b + c + d)(k + m)pAGD] \cdot \sigma(X,X)$$

$$\sigma(Y,Z) = (a + b + e + f)(a + b + c + d)AB \cdot \sigma(X,X) + (k + m) \cdot D \cdot \sigma(Y,Y)$$

$$\sigma(Y,R) = (a + b + c + d)(a + c + e + g)AC \cdot \sigma(X,X)$$
$$+ (j + k)F \cdot \sigma(Y,Y) + pG \cdot \sigma(Y,Z).$$

$$\sigma(Z,R) = (a + b + e + f)(a + c + e + g)BC \cdot \sigma(X,X) + (k + m)D(a + b + c + d) \cdot$$
$$(a + c + e + g)AC \cdot \sigma(X,X) + (j + k)F \cdot \sigma(Y,Z) + pG \cdot \sigma(Z,Z).$$

Deterministic Parameters	Probabilistic Pure-and Parameters
$\bar{A}$	$(a + b + c + d)A$
$\bar{B}$	$(a + b + e + f)B$
$\bar{C}$	$(a + c + e + g)C$
$\bar{D}$	$(k + m)D$
$\bar{F}$	$(j + k)F$
$\bar{G}$	pG

Table 2

The Mathematical Relations for the Probabilistic Interpretation of A. Jensen's Causal System (1980)

[Deterministic equations result when b = f = k = p = 1]

$$\sigma(X,R) = (b + d)B \cdot \sigma(X,X) + (f + h)D \cdot \sigma(X,Y)$$

$$\sigma(Y,R) = (b + d)B \cdot \sigma(X,Y) + (f + h)D \cdot \sigma(Y,Y)$$

$$\sigma(X,Z) = [(a + b)A + (b + d)(j + k)BH] \cdot \sigma(X,X) + [(e + f)C + (f + h)(j + k)DH] \cdot \sigma(X,Y)$$

$$\sigma(Y,Z) = [(a + b)A + (b + d)(j + k)BH] \cdot \sigma(X,Y) + [(e + f)C + (f + h)(j + k)DH] \cdot \sigma(Y,Y)$$

$$\sigma(X,S) = [(b + d)(k + m)BF + (a + b)pGA + (b + d)(j + k)pBGH] \cdot \sigma(X,X) + [(f + h)(k + m)DF + (e + f)pCG + (f + h)(j + k)pDGH] \cdot \sigma(X,Y)$$

$$\sigma(Y,S) = [(b + d)(k + m)BF + (a + b)pGA + (b + d)(j + k)pBGH] \cdot \sigma(X,Y) + [(f + h)(k + m)DF + (e + f)pCG + (f + h)(j + k)pDGH] \cdot \sigma(Y,Y)$$

$$\sigma(Z,R) = (a + b)(b + d)AB \cdot \sigma(X,X) + (f + h)(e + f)CD \cdot \sigma(Y,Y) + [(a + b)(f + h)AD + (b + d)(f + e)BC] \cdot \sigma(X,Y) + (j + k)H \cdot \sigma(R,R).$$

$$\sigma(S,R) = (k + m)F \cdot \sigma(R,R) + pG \cdot \sigma(Z,R)$$

$$\sigma(Z,S) = pG \cdot \sigma(Z,Z) + (k + m)F \cdot \sigma(Z,R).$$

Deterministic Parameters	Probabilistic Pure-and Parameters
$\bar{A}$	(a + b)A
$\bar{B}$	(b + d)B
$\bar{C}$	(e + f)C
$\bar{D}$	(f + h)D
$\bar{F}$	(k + m)F
$\bar{G}$	pG
$\bar{H}$	(j + k)H